

Theory for the use of foreign gas in simulating film cooling

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Abstract

In the film cooling of turbines the coolant is significantly cooler than the freestream. Consequently the coolant is at a higher density and this plays an important role in determining the flowfield. In laboratory experiments with small temperature differences this density difference is simulated by using dense foreign gas. This paper analyses the effect of molecular properties on the thermal measurements so that they may be related to the cold air situation. © 1999 Elsevier Science Inc. All rights reserved.

Keywords: Heat transfer; Film cooling; Foreign gas simulation

Notation

A, B, C	functions of position, Eq. (15)
C_p	specific heat at constant pressure
f_1, f_2	functions
$G_{x,y}$	mass flux, $\rho u_{x,y}$
i	specific enthalpy
k	thermal conductivity
M	mole fraction
m	mass fraction
Nu_x	Nusselt number = $\frac{qx}{k_g(T_{aw}-T_w)}$
Pr	Prandtl number
q	heat flux
Re_x	Reynolds number based on freestream and x
T	temperature
x	coordinate in freestream direction
y	coordinate perpendicular to surface
β	= $\left(\frac{\partial \beta}{\partial y}\right)_b$
δ	viscous sublayer thickness
ε	diffusivity
ζ	thermal diffusion coefficient
η	film cooling effectiveness
λ	mass diffusion coefficient
ξ	= $\epsilon\rho$
ρ	density
μ	viscosity
ϕ	= $-\left(\frac{\partial C}{\partial y}\right)_b$

Subscripts

A	cold air coolant
aw	adiabatic wall
b	evaluated at the bottom of the turbulent region
c	coolant
FG	foreign gas coolant
g	freestream
w	evaluated at the wall

1. Introduction

The use of foreign gas in film cooling experiments has been prevalent throughout the history of the subject. The foreign gas has been employed in the coolant flows for the following reasons:

1. To act as a tracer gas to determine the distribution of coolant (Goldstein, 1971).
2. To enable coolants of different density to be injected while at the same time acting as a tracer gas. The experiment is essentially isothermal (Goldstein, 1971; Pedersen et al., 1977).
3. To simulate coolants of different densities in heat transfer experiments (Teekaram et al., 1989).
4. To simulate coolants of different densities in aerodynamic experiments (Day et al., 1997).
5. There is a class of experiment in which a mass transfer to heat transfer analogy is employed (Eckert, 1986). The seminal paper on the application of the analogy to cases where there are temperature dependent properties is that of Shadid and Eckert (1991). Density is one of the temperature dependent properties and indeed it is the difference in temperature between coolant and freestream that leads to the density difference which is the subject of this paper. However, in the present paper heat transfer and temperature are actually measured and the mass to heat transfer analogy is not involved. The purpose in using foreign gas in the present paper is to generate approximately the same flowfield. The manner in which measured heat transfer and temperature are related to those in the situation where the density difference arises from temperature difference is the subject of this paper. Nevertheless, the adiabatic wall case may be considered as a form of the mass and heat transfer analogy as noted by Shadid and Eckert (1991) and as explained in the following Sections 3 and 4.

Combinations of these techniques are employed whereby the mass transfer analogy is used with a foreign gas injection

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(e.g. in Ammari et al., 1989) or a tracer gas is used within a different foreign gas (e.g. in Salcudean et al., 1994).

The objective of the present paper is to explore the interpretation of the third class of experiments above, referred to as case (3) in the following.

Although there are situations where gases dissimilar to the freestream are injected for film cooling purposes. This paper is concerned with the simulation of the gas turbine situation. In this case a significant ratio of coolant to freestream density occurs, resulting from their temperatures. The coolant temperature may be 1000 K and the freestream 2000 K giving density ratios of approximately two. In order to reproduce such large density ratios in a laboratory situation, recourse to the use of heavy foreign gas as the coolant has been common. At present, there is no general scaling parameter or procedure for the film cooling process and hence the need to simulate the coolant density. The first tests of the use of foreign gas to simulate the injection of relatively cold air coolant were those of Teekaram et al. (1989). In these experiments equivalent densities and mass flow rates were achieved by injecting CO₂ and air at different temperatures. The different thermal boundary conditions were taken into account by the normalisation of temperatures, inherent in the definition of film cooling effectiveness. Thus the mass flux and the momentum flux of coolant were reproduced and the assumption was that this generated the same flow field. At the same time little thought was given to the fact that the molecular properties of air and CO₂ were different, other than to note this difference and argue that the effects on the similarity of the parameters measured would be small. The experiments were performed to test this assumption and indeed showed good agreement between the air and CO₂ injection at the same coolant to freestream density ratio and injection to freestream velocity ratio. Since this time other workers have employed foreign gas to simulate density differences in heat transfer experiments (e.g. Han and co-workers, 1993, 1995, 1997).

In this paper analysis of the thermal field produced is undertaken. This results in a justification for the close correspondence in the CO₂/air experiments and also gives formulae to correct experimental data for different gases.

2. Simulation of coolant flows

The boundary layer flow field in gas turbine film cooling is highly turbulent. The transport of species, momentum and enthalpy is thus dependent on the turbulent flow field generated by the coolant injection. The role of viscosity is secondary other than in the surface sublayer and perhaps within the injection geometry. Thus, if coolant of the same density although a different gas, is injected with the same velocity then the turbulent flowfield should be similar. In the following therefore it is assumed that both the density ratio and velocity ratio of coolant to freestream is the same in the cold air and foreign gas simulation. Discharge coefficients of blade cooling holes are only Reynolds number-dependent at low flow rates (Rowbury et al., 1998). Also the thickness of a viscous sublayer under a flat plate turbulent boundary layer is dependent on Reynolds number to the tenth power. Hence the effects of Reynolds number changes due to foreign gas can be considered small in the first instance. Of course, freestream Reynolds number is assumed to be the same. It should be pointed out that the correspondence between flow fields referred to above obtains when both coolant density and mass flow (i.e. velocity) are simulated. Merely matching momentum flux is not a sufficient condition. Many workers do make the latter assumption and compare results for film cooling effectiveness and heat transfer

coefficient in this case. It is obvious that such comparisons are approximate at best.

3. Basic equations

The basic boundary layer differential equations for flows of gas mixtures are given by Kays and Crawford (1993). For a binary mixture of an inert, foreign gas coolant in a freestream gas the mass concentration equation is

$$G_x \frac{\partial m}{\partial x} + G_y \frac{\partial m}{\partial y} - \frac{\partial}{\partial y} \left(\lambda \frac{\partial m}{\partial y} \right) = 0. \quad (1)$$

The energy equation is

$$G_x \frac{\partial i}{\partial x} + G_y \frac{\partial i}{\partial y} - \frac{\partial}{\partial y} \left(\zeta \frac{\partial i}{\partial y} \right) = 0, \quad (2)$$

where m is the mass concentration of the coolant and i is the specific enthalpy of the mixture, i.e.

$$i = i_g(1 - m) + i_c m. \quad (3)$$

The local mass flux is G and λ and ζ are the mass and thermal diffusion coefficients, respectively. It has already been assumed that the diffusion coefficients are equal in arriving at these approximate governing equations. In the context of the present turbulent flow they refer to the turbulent quantities and their equality means that the turbulent Lewis number is unity. For gases this assumption is reasonable (Kays and Crawford, 1993).

The thermal problem must generally be specified in terms of enthalpy, i , boundary conditions rather than in terms of temperature, T , which is the conventional method. The use of temperature arrives at the usual description of film cooling in terms of the film cooling effectiveness, η , where

$$\eta = \frac{T_g - T_{aw}}{T_g - T_c},$$

suffices aw, g and c refer to the adiabatic wall, freestream and coolant, respectively. The effectiveness is independent of the temperature boundary conditions as may be shown by superposition (Jones, 1991).

In case (1) of experiments where a low concentration of tracer gas is employed the flow is essentially composed of a single gas and the energy equation may be written in terms of temperature.

$$G_x \frac{\partial T}{\partial x} + G_y \frac{\partial T}{\partial y} - \frac{\partial}{\partial y} \left(\zeta \frac{\partial T}{\partial y} \right) = 0. \quad (4)$$

The adiabatic wall case corresponds to that of an impermeable wall for the concentration equation. Thus at the wall (suffix w)

$$\left(\frac{\partial m}{\partial y} \right)_w \left(\frac{\partial T}{\partial y} \right)_w = 0. \quad (5)$$

The concentration field thus corresponds to the temperature field and

$$\frac{m}{m_c} \equiv \frac{T - T_g}{T_c - T_g} \quad (6)$$

giving the film cooling effectiveness as

$$\frac{m_w}{m_c} = \frac{T_{aw} - T_g}{T_c - T_g} = \eta. \quad (7)$$

In case (2) dense foreign gas is injected such that coolant density is significantly different to that of the freestream. This corresponds to the higher density of cold air injection. Again the majority of the flowfield and boundary conditions correspond for the adiabatic wall case and the film cooling effectiveness is given by Eq. (7). There is possibly an influence of

different properties in the sublayer on the flowfield due to the molecular viscosity and conductivity being different. However, Shadid and Eckert (1991) show that this effect is small.

4. Thermal field for an adiabatic wall

Case (3) considered in this section obtains when the foreign gas is at a different temperature to the freestream. The adiabatic wall temperature is determined in an experiment and the effectiveness for the equivalent air case is sought. In this case the enthalpy boundary conditions relate to the mass fraction as C_p is not constant. From Eq. (3)

$$C_p = C_{pg}(1 - m) + C_{pc}m. \quad (8)$$

In the following it is assumed that C_p 's of air and foreign gas are independent of temperature. Hence C_p is only a function of concentration. This assumption is valid for the foreign gas experiments as these are conducted at modest temperature differences, the large density differences being simulated by the concentration of foreign gas as already described. For the air or engine cases, large temperature differences are present, however, the value of C_p can be taken as being essentially constant. Thermal conductivity is dependent on temperature and the local value is used for the engine situation. Again, the thermal conductivity is a function of concentration for the foreign gas experiment.

At the wall

$$\left(\frac{\partial i}{\partial y}\right)_w = \left(\frac{\partial C_p}{\partial y}\right)_w T_w + \left(\frac{\partial T}{\partial y}\right)_w C_{pw} = C_{pw} \left(\frac{\partial T}{\partial y}\right)_w = 0$$

as the wall is impermeable and adiabatic.

Thus the boundary conditions for i and m are identical if the isothermal case at T_g is subtracted from the case considered as this is also a zero heat transfer situation. This is shown in the table below.

	enthalpy i	concentration m
Freestream	0	0
Coolant	$C_{pc}(T_c - T_g)$	1
Wall	$C_{pw}(T_{aw} - T_g)$	m_w
Thus	$\frac{C_{pw}(T_g - T_{aw})}{C_{pc}(T_g - T_c)} = \frac{m_w}{1}$	

but in this case $((T_g - T_{aw})/(T_g - T_c)) = \eta_{FG}$ i.e. the film cooling effectiveness measured with foreign gas. The film cooling effectiveness for cold air η_A with the same coolant density is given by setting $C_{pc} = C_{pg}$. Thus

$$m_w = \eta_A = \frac{C_{pw}}{C_{pc}} \eta_{FG}. \quad (9)$$

C_{pw} is found from Eq. (8), giving

$$\eta_{FG} = \frac{C_{pg}\eta_A}{C_{pg}(1 - \eta_A) + C_{pc}\eta_A} \quad (10a)$$

also

$$\eta_A = \frac{C_{pg}\eta_{FG}}{C_{pc}(1 - \eta_{FG}) + C_{pg}\eta_{FG}}. \quad (10b)$$

The suffices FG and A refer to the cases for foreign gas and air, respectively.

5. Heat transfer

The equivalence between the air and foreign gas enthalpy fields is not exact in case (3) when heat transfer is present as the

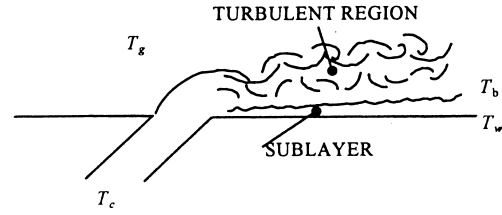


Fig. 1. Schematic diagram showing turbulent region and sublayer.

sublayer molecular properties will be influential and different. However, as in the previous section, it may be assumed that the turbulent flow field does correspond. If correspondence is thus sought for the turbulent region, the sublayer may be added subsequently. The temperature at the bottom of the turbulent layer is given by T_b as shown in Fig. 1.

The heat transfer at b in Fig. 1 is given by

$$q_b = \xi_b \left(\frac{di}{dy}\right)_b, \quad (11)$$

where $\xi_b = \epsilon_b \rho_b$. The quantities ϵ and ρ being the turbulent diffusivity and density, respectively. As $(\partial m/\partial y)$ at the wall is zero it can be assumed that

$$\left(\frac{\partial C_p}{\partial y}\right)_b = \left(\frac{\partial C_p}{\partial y}\right)_w = 0. \quad (12)$$

Thus

$$q_b = \epsilon_b \rho_b C_{pw} \left(\frac{dT}{dy}\right)_b. \quad (13)$$

Assuming that heat transfer takes place by conduction through the sublayer of thickness δ .

$$q_b = k_b \left(\frac{T_b - T_w}{\delta}\right), \quad (14)$$

where k is the conductivity.

The enthalpy field in the turbulent region will be of the form

$$i = A + BC_{pc}(T_c - T_g) + CC_{pw}(T_b - T_g), \quad (15)$$

where A , B and C are functions of position.

This expression arises due to the linearity of the energy equation, Eq. (2), in enthalpy allowing superposition of the enthalpy fields for different boundary conditions given in Fig. 2. A is the enthalpy field for isothermal conditions; B that for a coolant of unit enthalpy and zero enthalpy in the freestream and at the wall; C corresponds to unit enthalpy at the wall and zero freestream and coolant enthalpy.

Differentiating Eq. (15) with respect to y and using Eq. (11) gives the heat transfer rate as

$$\frac{q}{\xi_b} = \beta C_{pc}(T_c - T_g) - \phi C_{pw}(T_b - T_g), \quad (16)$$

where $\beta = (\partial B/\partial y)_b$ and $\phi = -(\partial C/\partial y)_b$.

When the wall is adiabatic then $T_b = T_{bad} = T_{aw}$. Thus

$$\left(\frac{T_g - T_{aw}}{T_g - T_c}\right) = \frac{C_{pc}\beta}{C_{pw}\phi} = \eta_{FG}.$$

When $C_{pc} = C_{pw} = C_{pg}$ this equals η_A and therefore

$$\eta_A = \frac{C_{pw}}{C_{pc}} \eta_{FG}.$$

This equation is the same as found previously, Eq. (9). Substituting back in Eq. (16) for $T_g - T_c$ gives

$$\frac{q}{\epsilon_b \rho_b} = \phi C_{pw}(T_{aw} - T_b). \quad (17)$$

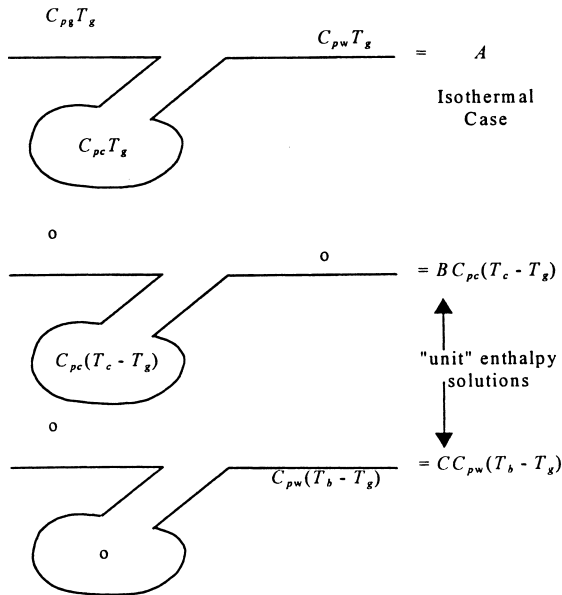


Fig. 2. Boundary conditions when superposed give the desired enthalpy boundary conditions of Fig. 1.

Eliminating T_b using Eq. (14) gives

$$\frac{q}{(T_{aw} - T_w)} = \frac{\phi \epsilon_b \rho_b C_{pw}}{1 + \frac{\phi \epsilon_b \rho_b C_{pw} \delta}{k_w}} \quad (18)$$

Thus the Nusselt number, Nu , becomes,

$$Nu = \frac{qx}{k_g(T_{aw} - T_w)} = \frac{\phi \epsilon_b \rho_b x}{k_g} \frac{C_{pw}}{1 + \frac{\phi \epsilon_b \rho_b \delta C_{pw}}{k_w}} \quad (19)$$

Dimensional reasoning shows that the term in Eq. (19) $\phi \epsilon_b \rho_b x$ may be expressed as a function of freestream Reynolds number and coolant injection properties

$$\frac{\phi \epsilon_b \rho_b x}{\mu_g} = f_1 \left(Re_{xg}, \frac{\rho_c}{\rho_g}, \frac{u_c}{u_g} \right).$$

The term containing δ may be similarly expressed in terms of function f_2 . Hence Eq. (19) becomes

$$Nu = \frac{Pr_g \frac{C_{pw}}{C_{pg}} f_1}{1 + Pr_g \frac{C_{pw}}{C_{pg}} \cdot \frac{k_g}{k_w} f_2} \quad (20)$$

Thus the ratio of Nusselt number for the cold air or engine case to that for the foreign gas simulation which is at the same Reynolds number and coolant to freestream density and velocity ratios becomes

$$\frac{Nu_A}{Nu_{FG}} = \left(\frac{C_{pg}}{C_{pw}} \right)_{FG} \frac{1 + \left[Pr_g \frac{C_{pw}}{C_{pg}} \frac{k_g}{k_w} \right]_{FG} f_2}{1 + \left[Pr_g \frac{k_g}{k_w} \right]_A f_2} \quad (21)$$

Again it is assumed that C_p and Pr for air are approximately independent of temperature.

Analytical solutions for constant property flows matching the viscous sublayer to the turbulent region above are available and may be used to obtain an approximate value of f_2 .

Thus an estimate of the flowfield term f_2 in Eq. (21) may be made from the heat transfer to a flat plate in a uniform flow. From standard texts (e.g. Kays and Crawford, 1993) the constant property equation, for gases, is

$$Nu_x = \frac{0.0287 Re_x^{0.8} Pr}{0.169 Re_x^{-0.1} (13.2 Pr - 8.66) + 0.85}, \quad (22)$$

where Pr is the Prandtl number and Re_x is the Reynolds number based on distance x , from the leading edge. Eq. (22) may be rearranged to give

$$Nu_x = \frac{Pr f_1(Re_x)}{1 + \frac{2.62 Pr}{Re_x^{0.1} - 2.00}} \quad (23)$$

The last term in the denominator corresponding to the thermal resistance of the sublayer represented by the term $f_2 Pr$ in Eq. (20) and Eq. (21). As can be seen the term is relatively insensitive to Reynolds number and as stated refers to the constant property case.

$$f_2 = \frac{2.62}{Re_x^{0.1} - 2.00} \quad (24)$$

For $Re_x = 10^5$, $f_2 = 2.25$ and for $Re_x = 10^6$, $f_2 = 1.32$ C_{pwFG} and k_{wFG} are determined from $\eta_A = m$ found from Eq. (10b). C_{pwFG} is given by Eq. (8). k_{wFG} is approximately given by an equation of the same form, i.e.

$$k = k_g(1 - M) + k_c M \quad (25)$$

but in this case M is the molar concentration determined from m . The value of k for the air case is determined by the relevant temperature.

It should finally be mentioned that the foregoing analysis has assumed that the local wall temperature represents the isothermal wall situation. This is not exact, however, as Eckert (1992) points out "the upstream temperature or heat flux distribution has little effect on the local Stanton number in a turbulent boundary layer".

6. Experimental results

Heat transfer results are available for carbon dioxide injection and for injection using a gas mixture of sulphur hexafluoride and argon. The mixture has the property that its ratio of specific heats is 1.4 i.e. the same as that of air. In this manner compressibility effects should also be reproduced although the analysis in this paper does not include compressibility. The properties are given in Table 1.

Eq. (10b) gives the air film cooling effectiveness as a function of the measured foreign gas effectiveness and this is plotted in Fig. 3 for CO_2 and $SF_6 + A$ mixtures. There is little correction for the CO_2 injection whereas the correction is significant for $SF_6 + A$. The experimental results of Teekaram et al. (1989) for CO_2 injection are given in Fig. 4. These are compared with experimental results for the same injection density and velocity ratios but with air as the coolant. As can be seen the results are extremely close. In general the results for the CO_2 are slightly below that for air as would be predicted by Eq. (10b). However the experimental error obscures close analysis of the difference.

Examples of the correction necessary to give engine film cooling effectiveness and heat transfer coefficient are given in Figs. 5 and 6 from Guo et al. (1998). In these cases $SF_6 + A$ was the foreign gas employed. The correction can be seen to be significant.

7. Conclusions

An approximate procedure has been set out for the correction of film cooling experimental results which employ

Table 1
Properties of air, CO₂ and SF₆ + A mixtures (air and SF₆ + A from Oldfield and Guo (1997))

Gas	Density ratio ρ_{FG}/ρ_A	C_p (kJ kg ⁻¹)	k (W m ⁻¹ K ⁻¹)	Pr	μ (kg m ⁻¹ s ⁻¹)
air (300 K)	1.0	1.005	2.614×10^{-2}	0.711	1.85×10^{-5}
CO ₂ (300 K)	1.52	0.852	1.662×10^{-2}	0.768	1.499×10^{-5}
SF ₆ + A (300 K)	1.77	0.564	1.67×10^{-2}	0.733	2.17×10^{-5}
air (1000 K)	–	1.141	6.72×10^{-2}	0.709	4.177×10^{-5}
air (1800 K)	–	1.286	0.111	0.701	6.07×10^{-5}

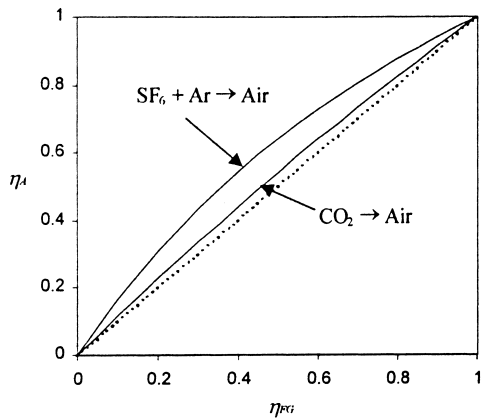


Fig. 3. Film cooling effectiveness for cold air, η_A , versus the equivalent value for foreign gas, η_{FG} . Eq. (10b). (b) Density ratio $\rho_c/\rho_g = 1.67$.

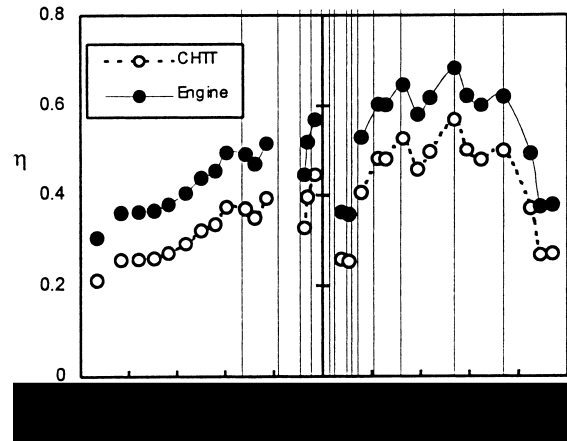
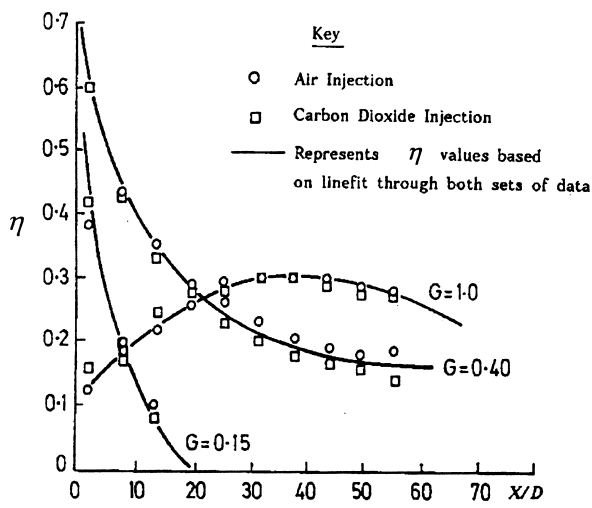


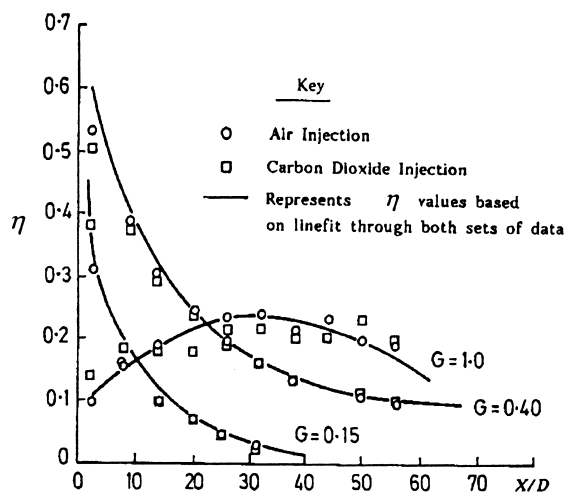
Fig. 5. Film cooling effectiveness, η , on an NGV determined from experiments using SF₆ + A as the coolant (CHTT) and the value determined for the engine using Eq. (10b). The lines show the positions of rows of film cooling holes.

foreign gas to achieve large coolant densities compared to the freestream. This takes into account the difference in specific heat and thermal conductivity. The correction has been shown to agree with the experimental observation that

there is little correction necessary in the case of CO₂ injection. In general both film cooling effectiveness and Nusselt number require correction according to the formulae presented.



(a) Density ratio $\frac{\rho_c}{\rho_g} = 1.25$



(b) Density ratio $\frac{\rho_c}{\rho_g} = 1.67$

Fig. 4. Results for effectiveness η versus distance X/D downstream from a single row of 30° holes of diameter D . Carbon dioxide and air injection results for the same density ratios. From Fig. 3, it can be seen that the results for air are expected to be slightly above those for CO₂.

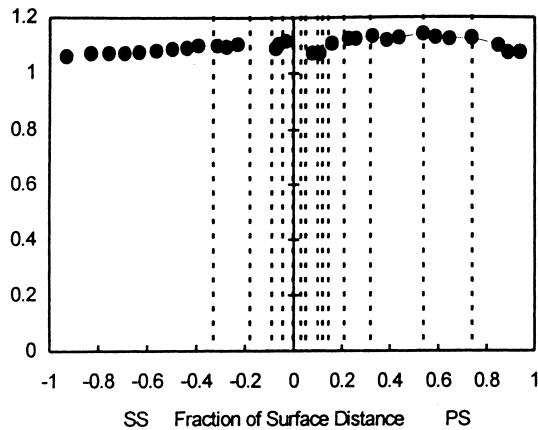


Fig. 6. The ratio of Nusselt number for the engine to that determined for SF₆ + A coolant in an experiment on an NGV from Eq. (21). Dotted lines are rows of coolant injection holes.

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